

GCE

Further Mathematics B (MEI)

Y433/01: Modelling with algorithms

Advanced GCE

Mark Scheme for Autumn 2021

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

Question			Answer	Marks	AOs	Guidance
1	(a)		Bin 1: 5 16 12 10 Bin 2: 15 21 5 3 Bin 3: 17 6 13 5 Bin 4: 24	M1 A1 [2]	1.1 1.1	First six values placed correctly (the values in bold) – so the 10 in the correct bin cao
	(b)		e.g. Bin 1: 24 21 Bin 2: 16 15 6 5 3 Bin 3: 17 13 10 5 Bin 4: 12 5	M1 A1 [2]	1.1 1.1	At least two full bins (= 45) cao (three full bins with 17 units in the non-full bin)
	(a) (b)			M1 A1 A1 [3]	3.1b 3.1a 1.1	Activity on arc, single start vertex Precedences correct for A, B, C, D, G, H Directions may be implied Durations not necessary Single finish Precedences correct for E, F, I, J, K Directions may be implied Durations not necessary All three dummies correct and no extras All arcs directed
				M1 ft M1 ft A1 [3]	3.1b 1.1 1.1	Network must have at least one burst and at least one merge, other than start and finish Forward pass, increasing, allow 1 blank Backward pass, decreasing, allow 1 blank Forward pass and backward pass correct

Question			Answer	Marks	AOs	Guidance
2	(c)		Minimum completion time is 31 (hours)	B1ft [1]	2.2a	Follow through their network
2	(d)		Interfering float for H is $(22 - 8) - (21 - 8) = 1$ (hour)	B1ft [1]	3.4	Follow through using their early and late event times at the beginning and end of H
2	(e)		Total float for E is $21 - 11 - 6 (= 4)$ and Total float for G is $21 - 8 - x (= 13 - x)$ $13 - x \leq 2 \times 4$ or $13 - x \leq 8$ $5 \leq x < 13$	M1 * M1dep* A1 [3]	1.1 2.1 2.2a	Correct calculations of the total float for their E and G Using the given information to set up an inequality for x cao
3	(a)	(i)	The sum of the vertex orders equals the number of arc endings Each arc has two ends so the number of arc endings is twice the number of arcs So the sum of the vertex orders is twice the number of arcs, which is even	B1	2.1	States or uses the result that the sum of the order of the vertices is equal to twice the number of arcs
			Alternative method Let a graph have e edges and n nodes (vertices), let d_i represent the order of the i th node so $\sum_{i=1}^n d_i = 2e$, which is even	B1		
				[1]		
3	(a)	(ii)	The sum of the orders of all the even vertices will be an even number so the sum of the order of the odd vertices must be an even number too Hence a graph must have an even number of vertices of odd order So no graph has an odd number of odd vertices	B1 [1]	2.2a	Correctly explains why a graph cannot have an odd number of vertices with odd order (or must have an even number of vertices with odd order) Must refer to even vertices as well as odd

Question			Answer	Marks	AOs	Guidance																											
3	(b)		Shortest path from A to F is ACDHIF	M1 A1 A1 A1 B1 [5]	1.2 1.1a 1.1a 1.1 1.1	Correct working values at D Working values Labels Order of labelling Allow one slip																											
3	(c)		STEP 1 <table><thead><tr><th>Possible pairings of odd nodes</th><th>Corresponding shortest path</th><th>Weight of shortest path</th></tr></thead><tbody><tr><td>AE</td><td>ACBE</td><td>37</td></tr><tr><td>AG</td><td>ACBG</td><td>32</td></tr><tr><td>AI</td><td>ACDHI</td><td>48</td></tr><tr><td>EG</td><td>EBG</td><td>25</td></tr><tr><td>EI</td><td>EBGI</td><td>43</td></tr><tr><td>GI</td><td>GI</td><td>18</td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></tbody></table>	Possible pairings of odd nodes	Corresponding shortest path	Weight of shortest path	AE	ACBE	37	AG	ACBG	32	AI	ACDHI	48	EG	EBG	25	EI	EBGI	43	GI	GI	18							M1 * M1 dep* M1 dep* A1 [4]	1.1 1.1 1.1 1.1	Any two rows correct Any three rows correct Any four rows correct All correct
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Question			Answer	Marks	AOs	Guidance
3	(d)		STEP 2 AE and GI	B1	3.4	Both chosen, allow ACBE and GI
			STEP 3 $353 + 37 + 18 = 408$	B1 [2]	1.1	cao
4	(a)	(i)	Cut $\alpha = 22 + 43 + 71 + 47 = 183$	B1	1.1	cao, need not show working
				[1]		
4	(a)	(ii)	Cut $\beta = 82 + 33 + 43 + 71 + 25 + 39 = 293$	B1	1.1	cao, need not show working
				[1]		
4	(b)		The maximum possible flow is (at most) 183 (litres per minute)	B1 ft [1]	1.1	$\min\{\text{their (a)(i), their (a)(ii)}\}$
4	(c)		The only arc leading into C is SC and the only arcs out of C are CB and CF and hence $SC - CB - CF = 0$	B1 [1]	2.4	Flow in = flow out at C and stating that these are the only arcs that flow into C and out of C
4	(d)		Maximise $DT + ET + GT$	B1	3.1b	Maximise and $DT + ET + GT$
			$SB + AB + CB - BD - BE - BG - BF = 0$ $BE + DE - EG - ET = 0$	B1	3.3	Flow in = flow out at B and at E represented using these equations
			$DT \leq 82, ET \leq 24, GT \leq 67$	B1 [3]	3.3	Capacities for arcs into T represented using these inequalities

Question		Answer	Marks	AOs	Guidance
4	(e)		M1 A1 [2]	2.1 2.2a	Flow = 152. Consistent flow pattern (flow in = flow out at each node) – flow through every arc apart from DE and EG Condone incorrect or missing flow through one arc for the M mark A correct flow (flow $\leq$ capacity for each arc)
4	(f)	The capacity of the cut which partitions the vertices into the sets $\{S, A, B, C, E, F, G\}$, $\{D, T\}$ is $22 + 39 + 24 + 67 = 152$ [$\therefore$ minimum cut is ≤ 152] By the maximum flow-minimum cut theorem the maximum flow is equal to the minimum cut and so therefore the maximum flow through the system is 152 litres per minute	M1 A1 [2]	3.1b 2.1	$\{S, A, B, C, E, F, G\}$, $\{D, T\}$ described in any way (but not implied) Max flow = min cut (o.e)
4	(g)	From the source there is only one non-saturated arc SA and into the sink there is only one non-saturated arc DT. Therefore the flow can be increased by the least of $82 - 61 = 21$ and $62 - 34 = 28$ giving a maximum flow of $152 + 21 = 173$ (litres per minute) The corresponding value of x is $21 + 22 = 43$	B1 B1 [2]	3.4 2.2a	173 43

Question			Answer	Marks	AOs	Guidance																																																																																																
5	(a)		$x + y + z = 50 \Rightarrow x + y + z \leq 50$ and $x + y + z \geq 50$	M1	3.1a	Dealing with equality constraint as two inequalities or implied from two correct equations (with slack, surplus and artificial variables)																																																																																																
			$x + y + z + s_1 = 50$ and $x + y + z - s_2 + a_1 = 50$	A1	1.1	Or SC B1 for one correct equation (if previous mark not earned)																																																																																																
			$x \leq 25 \Rightarrow x + s_3 = 25$	M1	1.1	Adding a slack variable appropriately to any of these three																																																																																																
			$-y + 3z \leq 0 \Rightarrow -y + 3z + s_4 = 0$	A1	3.1b	All three correct in this form Allow $x - y - z \leq 0$ o.e. for $x \leq 25$ Or equivalent with surplus and artificial variables in one of these equations																																																																																																
			$x + 4y + 12z \leq 210 \Rightarrow x + 4y + 12z + s_5 = 210$																																																																																																			
			$P = 2x + 5y + 20z \Rightarrow P - 2x - 5y - 20z = 0$	B1	3.1a	cao																																																																																																
			$Q = a_1$ so $Q + x + y + z - s_2 = 50$	M1	2.1	Attempt to substitute expression for a_1 (artificial variable for equality constraint)																																																																																																
				A1	2.2a	cao																																																																																																
			<table><tr><th>Q</th><th>P</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>s_5</th><th>a_1</th><th>RHS</th></tr><tr><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>50</td></tr><tr><td>0</td><td>1</td><td>-2</td><td>-5</td><td>-20</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>50</td></tr><tr><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>50</td></tr><tr><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>25</td></tr><tr><td>0</td><td>0</td><td>0</td><td>-1</td><td>3</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td><td>1</td><td>4</td><td>12</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>210</td></tr></table>	Q	P	x	y	z	s_1	s_2	s_3	s_4	s_5	a_1	RHS	1	0	1	1	1	0	-1	0	0	0	0	50	0	1	-2	-5	-20	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	50	0	0	1	1	1	0	-1	0	0	0	1	50	0	0	1	0	0	0	0	1	0	0	0	25	0	0	0	-1	3	0	0	0	1	0	0	0	0	0	1	4	12	0	0	0	0	1	0	210	M1	3.3	Any three rows correct
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Question	Answer	Marks	AOs	Guidance
5 (b)	$x \leq 25, -y + 3(50 - x - y) \leq 0$ $\text{and } x + 4y + 12(50 - x - y) \leq 210$ $x \leq 25, 3x + 4y \geq 150 \text{ and } 11x + 8y \geq 390$ 	M1 * M1 dep* A1 M1 A1 [5]	3.1a 1.1 1.1 1.1 1.1	Substitute $x + y + z = 50$ to form expressions in x and y only Any two of these correct All correct, need not be simplified Two of their lines drawn correctly (may need to check constraints in (a) as well) All three lines correct with correct feasible region, from shading or labelled

Question			Answer	Marks	AOs	Guidance
5	(c)		$P = 2x + 5y + 20(50 - x - y) \Rightarrow P = (1000) - 18x - 15y$ So maximising the negative expression $-3(6x + 5y)$ is equivalent to minimising the equivalent positive expression $3(6x + 5y)$ and the optimal values of x and y can be found by just considering $6x + 5y$	M1 A1 [2]	3.4 2.4	Substitute $x + y + z = 50$ into P and simplify
5	(d)	(i)	Leo should answer 18 algebra questions, 24 trigonometry questions and 8 calculus questions	B1 [1]	3.2a	In context
5	(d)	(ii)	Leo will score 316 points	B1 [1]	1.1	
5	(e)		There is no guarantee that Leo will get the answers to the questions correct	B1 [1]	3.5b	oe correct reason

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